Examples:

1). Find the value of $\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

Let the value we are being asked to find equal x.

Then $x = \sqrt{7+x}$

$x^2 - x - 7 = 0$

Using the quadratic formula,

$x = \frac{1 \pm \sqrt{29}}{2}$

We know that the positive solution is correct because the square roots in the original expression imply it.

2). Compute: $1 + \frac{2}{1 + \frac{2}{1+\ldots}}$

Let this value equal a.

Then,

$a = 1 + \frac{2}{a}$

$a^2 - a - 2 = 0$

$(a-2)(a+1) = 0$

$a = 2$

the solution $a = -1$ is extraneous because the original equation was positive.

3). Find $\frac{1}{4} + \frac{3}{20} + \frac{9}{100} \ldots$

Let this value equal s.

Then $\frac{3}{20} + \frac{9}{100} \ldots = \frac{3s}{5}$ (from subtracting the two equations)

$s - \frac{3s}{5} = \frac{1}{4}$

$\frac{2s}{5} = \frac{1}{4}$

$s = \frac{5}{8}$