SIMC 5th Grade: Week 3

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1 Speed Math: Multiplying Numbers Around 100

This method takes advantage of the ease of multiplying numbers by 100.

Directions for multiplying numbers a, b (generally $90 \leq a, b \geq 100$)

- Subtract 100 from one of the numbers to get $a - 100$.
- Add the result to the other number to get $a + b - 100$.
- Multiply this number by 100 to get $100a + 100b - 100^2$.
- Separately, subtract 100 from each of the numbers to get $a - 100$ and $b - 100$.
- Multiply the two values to get $ab - 100a - 100b + 100^2$.
- Add this to your previous total to get ab!

Example: Multiply 92 by 103

- First, we choose one of the numbers (103) and subtract 100 to get 3.
- Then, we add this to the other number to get 95.
- Multiplying this number by 100 to get 9500.
- Separately, we subtract 100 from each of the two numbers to get -7 and 3.
- We multiply these together to get -21.
- We add this to our previous total to get $9500 - 21 = 9479$ as our answer.

2 Rolling Boulders Up a Hill

Examples

1. **Problem:** Eho is trying to move a large boulder up a hill. He can move it up 5 feet per day, but Biff moves the boulder down 2 foot each night. How long will it take Eho to move the boulder up the 125 foot hill?

   Solution: Subtract 5 from the 125 to get 120. Then, divide 120 by $5 - 2 = 3$ to get 40. Add back 1 for the 5 feet you subtracted away to get a final answer of 41. It was important to subtract away the 5 first because Biff doesn't move the boulder down any more if Eho has already reached the top of the hill.

2. **Problem:** Eho starts to weld together the pieces to build his own ship. Every piece must be welded to every other piece. Eho can make 12 welds per day, but Biff will cut apart 5 of his welds each night. How long will it take Eho to build his ship if he has 20 pieces?

   Solution: Eho must make $\frac{20x19}{2} = 190$ welds. Subtract 12 from 190 to get 178 welds left. Then, divide 178 by $12 - 5 = 7$ to get approximately 26 (if not a whole number, round up). Add make one more to get a final total of 27 days to complete the project.

General Solution

General Problem: Suppose that Eho can complete x tasks each day. Then, Biff undoes y of those tasks during the day. We are being asked how long it will take Eho to complete T tasks.

Solution:

1. First, we must subtract off x from T because, once Eho has completed at least $T - x$ tasks, then he will be able to complete the remainder of the tasks the next day, finishing all of them before Eho has a chance to undo any.
2. Then, we can divide what's left by $x - y$, to get $\dfrac{T - x}{x - y}$
3. We must round this up. Our result it $\lfloor \dfrac{T - x}{x - y} \rfloor$
4. Finally, we add back 1 more day to compensate for the x that we took off at the very beginning.

5. Our final result after we combine terms is $\lfloor \frac{T-y}{x-y} \rfloor$

3 Roots of a Tree: Polynomial Definitions and Notation

We generally write polynomials in the form

$$f(x) = a_0 x^n + a_1 x^{n-1} \ldots + a_{n-1} x + a_n$$

where $a_0, a_1 \ldots a_n$ are coefficients (numbers) and x is a variable.
Note that all polynomials are functions, but not all functions are polynomials. Polynomials can be written in different forms, but they always come in this style. There is no xy term in a polynomial.

F can be replaced by any other letter, symbol, etc (some of the most common are g and h). $f(x)$ for a particular x denotes the return value when that value of x is plugged into the function. The degree of a polynomial is the highest power of x, namely n. A polynomials of degree n will have exactly n non-distinct roots.

A **root** of a value for x such that the return value of $f(x)$ is 0. The roots of a polynomials are generally written $r_1, r_2, r_3, \ldots r_n$ for a polynomial of degree n.

The form given above is the **expanded** form of the polynomial. Using summation notation, this an also be written as $\sum n = 0^d a_n x^{d-n}$ where d is the degree of the polynomial.

We write the **factored** form of a polynomial as
$f(x) = (x - r_1)(x - r_2) \ldots (x - r_n)$. This can also be written as
$\prod n = 1^d (x - r_n)$.