SEATTLE INFINITY MATH CIRCLE

Mock MathCounts Test 2011

Written by: Albert Chu

MATHCOUNTS

Sprint Round

Name: _____

Score: _____

1. What is greatest common divisor (GCD) of 2011 and 20011? 1. _____

2. Convert $0.1\overline{3}$ into a common fraction. 2. _____

3. What is the smallest 4-digit palindrome number that is a multiple of 7? 3. _____

4. Two standard six-sided die will be rolled 20 times. Let $p_k(S)$ be the probability of the sum of two numbers in the die is equal S at the k-th roll. What is product of $p_3(12) + p_{12}(3)$? 4. _____

5. A car is scheduled to make a 700-mile trip in 10 hours. The car averages 60 mph during the first 240 miles and 80 mph during the next 160 miles. What must the average speed (miles per hour) of the car be for the remainder of the trip in order for the car to arrive on schedule? 5. _____ mph

6. Three consecutive even integers $a < b < c$ satisfy $c^2 - a^2 = 128$. Find the value of b. 6. _____

7. Albert has \$11.07 in his pocket. He is very sure that he has an equal number of pennies, nickels, dimes and quarters. How many of each type of coin does he has? 7. _____ coins

8. If a and b are two positive integers and $ab = 256$, what is the smallest possible value of $a + b$? 8. _____

9. Find the smallest 4-digit number that has all different digits and is divisible by each of its digits.

9. _____

10. Consider an extremely large number $n = 10^{2011} + 8$. Note that n is a multiple of 9, that is, $n = 9k$, where k is a natural number. What is the sum of all the digits of the natural number k?

10. _____

11. Consider another extremely large number $n = 2011!$ In how many zeroes does this number n end?

11. _____ zeroes

12. If 10 men take 6 days to lay 1000 bricks, then how many days will it take 30 men working at the same rate to lay 5000 bricks?

12. _____ days

13. Albert has a key ring that has 7 keys. How many distinct ways can he arrange these 7 keys on the key ring?

13. _____ ways

14. If the product of three positive integers x, y, z is 40, then how many such ordered triplets (x, y, z) exist?

14. _____ triplets

15. A parallelogram has three known vertices $(0,0)$, $(3,3)$, $(-3,3)$, and an unknown vertex (x, y). Let S_x be the sum of all possible values of x and S_y be the sum of all possible values of y. What is the value of the multiplication $S_x S_y$?

15. _____

16. A triangle has side lengths 13, 5, and 12. What is the area of this triangle?

16. _____ sq units

17. What is the sum of the unit digits of n^4 for $n = 1, 2, 3, \ldots, 30$? 17. _____

18. What are the last two digits of 31^{42} ? 18. _____

19. What is the smallest whole number that has a remainder of 1 when divided by 4, a remainder of 1 when divided by 3, and a remainder of 2 when divided by 5? 19. _____

20. The graphs $y = |x|$ and $y = 10 - |x|$ enclose a region R. Find the area of region R. 20. _____ sq units

21. The sum of a particular set of three consecutive positive integers is a three-digit palindrome. If the sum is less than 290, what is the greatest possible value of this sum? 21. _____

22. How many perfect squares less than 1,000 have 6, 7, or 8 as the units digit? 22. _____

23. An integer function $f(x)$ is valid only for $x = 0,1,2,3$ and has an interesting property $f(f(x)) = x$. It is also known that $f(x) \neq x$. Find out how many such functions exist. 23. _____

24. Let a and b be different digits such that $a < b < 6$. Consider a 9-digit number $abb, aba, ba3$ that is a multiple of 99. Find the difference between a and b.

24. _____

25. A very large natural number contains 4001 digits: 1000 digit 3's, 3000 digit 2's, and unit digit 5. What is the remainder of this very large number divided by 6?

25. _____

26. There are two parallel lines: $2x + 3y = 10$ and $2x + 3y = 20$. What is the distance between these two lines?

26. _____

27. A $4 \times 4 \times 4$ cubes are painted outside. Let N_0, N_1, N_2, and N_3 be the numbers of the cubes without paint, painted 1 side, 2 sides, and 3 sides, respectively. What is the value of $\sqrt{N_0 N_1 N_2 N_3}$?

27. _____

28. Four neighboring vertices of a unit cube form a pyramid. What is the volume of this pyramid?

28. _____ cubic units

29. A parallelogram has three known vertices $(7,1)$, $(5,-3)$, and $(-3,1)$. The fourth vertex is unknown. What is the average value of the areas of all possible such parallelograms?

29. _____ sq. units

30. How many ways to color a cube using 6 colors, where each face has a unique color?

30. _____ ways